Rotational Motion: every formula

Keep this beside you while you watch the one-shot. Same order as the lecture.

1 · Centre of mass

Two bodies	$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$	Many bodies	$x_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i}$
Distances	$m_1r_1 = m_2r_2$ (nearer the heavier)	Rod, ring, disc	at the geometric centre
Semicircular ring	$\frac{2R}{\pi}$ from the centre	Semicircular disc	$\frac{4R}{3\pi}$ from the centre
Solid hemisphere	$\frac{3R}{8}$ from the centre	Cut-out	$x = \frac{M_1x_1 - M_2x_2}{M_1 - M_2}$
CM velocity	$v_{cm} = \frac{\Sigma m_i v_i}{\Sigma m_i}$	No external force	v_{cm} stays constant

2 · Rotational kinematics

Angular velocity	$\omega = \frac{d\theta}{dt}$	Angular acc.	$\alpha = \frac{d\omega}{dt}$
Linear speed	$v = \omega r$	Tangential acc.	$a_t = \alpha r$
Centripetal acc.	$a_c = \omega^2 r = \frac{v^2}{r}$	Net acc.	$a = \sqrt{(a_t^2 + a_c^2)}$
Equation 1	$\omega = \omega_0 + \alpha t$	Equation 2	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
Equation 3	$\omega^2 = \omega_0^2 + 2\alpha\theta$	Units	rpm $\times \frac{2\pi}{60}$ = rad/s

3 · Torque and equilibrium

Torque	$\tau = r \times F$	Its size	$\tau = rF \sin\theta = F \cdot r_\perp$
Direction	right-hand rule	Couple	$\tau = F \cdot d$, same about any point
Equilibrium	$\Sigma F = 0$ and $\Sigma\tau = 0$	Balance a rod	torques about the pivot
Work by torque	$W = \tau\theta$	Power	$P = \tau\omega$

4 · Angular momentum

Particle	$L = r \times p$	Its size	$L = mvr_\perp = mvr \sin\theta$
Rigid body	$L = I\omega$	Newton's 2nd law	$\tau = \frac{dL}{dt}$
Conservation	$\tau_{ext} = 0 \Rightarrow I_1\omega_1 = I_2\omega_2$	Skater, arms in	$I \downarrow \Rightarrow \omega \uparrow$, KE $\uparrow$
Two discs join	$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$	When they stick	L conserved, KE lost
KE and L	$KE = \frac{L^2}{2I}$		

5 · Moment of inertia

Ring, ⊥ axis	MR^2	Ring, diameter	$\frac{1}{2}MR^2$
Disc, ⊥ axis	$\frac{1}{2}MR^2$	Disc, diameter	$\frac{1}{4}MR^2$
Hollow cylinder	MR^2	Solid cylinder	$\frac{1}{2}MR^2$
Solid sphere	$\frac{2}{5}MR^2$	Hollow sphere	$\frac{2}{3}MR^2$
Rod, centre	$\frac{ML^2}{12}$	Rod, one end	$\frac{ML^2}{3}$
Point mass	mr^2	Any body	$I = \Sigma mr^2$

6 · Axis theorems

Parallel axis	$I = I_{cm} + Md^2$	Perpendicular axis	$I_z = I_x + I_y$ (flat body)
Radius of gyration	$I = Mk^2,\ k = \sqrt{(I/M)}$	Ring, tangent ∥	$\frac{3}{2}MR^2$
Ring, tangent ⊥	$2MR^2$	Disc, tangent ∥	$\frac{5}{4}MR^2$
Disc, tangent ⊥	$\frac{3}{2}MR^2$	Sphere, tangent	$\frac{7}{5}MR^2$
Shell, tangent	$\frac{5}{3}MR^2$	Smallest I	axis through the CM

7 · τ = Iα and energy

Newton's law	$\tau = I\alpha$	Rotational KE	$KE = \frac{1}{2}I\omega^2$
Work-energy	$W = \Delta KE$	Rope on a wheel	$\tau = TR,\ a = \alpha R$
Mass on a wheel	$a = \frac{mg}{m + I/R^2}$	Hinged rod, let go	$\alpha = \frac{3g}{2L}$
Same ω	$KE \propto I$	Same L	$KE \propto \frac{1}{I}$

8 · Rolling

No slipping	$v = \omega R$	Top / contact point	2v / 0
Total KE	$\frac{1}{2}mv^2(1 + \frac{k^2}{R^2})$	k^2/R^2	ring 1, shell ⅔, disc ½, sphere ⅖
Spin share of KE	$\frac{k^2/R^2}{1 + k^2/R^2}$	Down a slope	$a = \frac{g \sin\theta}{1 + k^2/R^2}$
Speed at bottom	$v^2 = \frac{2gh}{1 + k^2/R^2}$	Race winner	smallest k^2/R^2
Friction	no work in pure rolling		

Translation → rotation: one table rebuilds every formula

x	→	θ	v	→	ω
a	→	α	m	→	I
F	→	τ	$p = mv$	→	$L = I\omega$
$F = ma$	→	$\tau = I\alpha$	$\frac{1}{2}mv^2$	→	$\frac{1}{2}I\omega^2$
$P = Fv$	→	$P = \tau\omega$			

10 traps NEET sets again and again

1. Every point of a rigid body has the SAME ω
2. Convert first: rpm → rad/s, revolutions → radians
3. A couple has the same torque about every point
4. Perpendicular axis theorem: flat bodies only
5. Cut-out: subtract I about the SAME axis
6. Ratio of k needs a square root of the I ratio
7. Same ω → KE $\propto$ I; same L → KE $\propto$ 1/I
8. Discs that stick: L conserved, KE lost
9. Rolling: top point 2v, contact point 0
10. Rolling race: only k^2/R^2 matters